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*Published in:*  
Journal of High Energy Physics

*DOI:*  
[10.1007/JHEP05\(2018\)136](https://doi.org/10.1007/JHEP05(2018)136)

*Publication date:*  
2018

*Document version*  
Publisher's PDF, also known as Version of record

*Document license:*  
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*Citation for published version (APA):*  
Brivio, I., & Trott, M. (2018). Addendum to: Scheming in the SMEFT... and a reparameterization invariance! . *Journal of High Energy Physics*, 2018(5), [136]. [https://doi.org/10.1007/JHEP05\(2018\)136](https://doi.org/10.1007/JHEP05(2018)136)

## Addendum: Scheming in the SMEFT... and a reparameterization invariance!

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ADDENDUM TO: [JHEP07\(2017\)148](#)

ABSTRACT: We allow two independent flavor contractions for the operator  $Q_{ll}$  in the  $U(3)^5$  flavor symmetric limit and report modified fit results in this limit.

KEYWORDS: Effective Field Theories, Beyond Standard Model

ARXIV EPRINT: [1701.06424](#)

The original paper adopted an overly restrictive form of a  $U(3)^5$  limit by not allowing two independent flavor contractions admitted by the operator  $\mathcal{Q}_{ll}$  in the  $U(3)^5$  flavor symmetric limit [2]. Defining

$$\mathcal{L}_{\text{SMEFT}} \supset [C_{ll} \delta_{mn} \delta_{op} + C'_{ll} \delta_{mp} \delta_{no}] (\bar{l}_m \gamma_\mu l_n) (\bar{l}_o \gamma^\mu l_p),$$

both  $C_{ll}$  and  $C'_{ll}$  are allowed to be independent parameters in the  $U(3)^5$  flavour symmetric limit. The original paper used the same parameter  $C_{ll}$  in both terms, which is overly restrictive. This leads to  $C_{ll} \rightarrow C'_{ll}$  in the expressions:

$$\delta G_F = \frac{1}{\sqrt{2} \hat{G}_F} \left( \sqrt{2} C_{H\ell}^{(3)} - \frac{1}{\sqrt{2}} C'_{ll} \right), \quad (4)$$

$$\delta \bar{g}_Z = -\frac{1}{\sqrt{2}} \delta G_F - \frac{1}{2} \frac{\delta m_Z^2}{\hat{m}_Z^2} + \frac{s_{\hat{\theta}} c_{\hat{\theta}}}{\sqrt{2} \hat{G}_F} C_{HWB} = -\frac{1}{4\sqrt{2} \hat{G}_F} \left( C_{HD} + 4 C_{H\ell}^{(3)} - 2 C'_{ll} \right), \quad (15)$$

$$\delta g_1^\gamma = \frac{1}{4\sqrt{2} \hat{G}_F} \left( C_{HD} \frac{\hat{m}_W^2}{\hat{m}_W^2 - \hat{m}_Z^2} - 4 C_{H\ell}^{(3)} + 2 C'_{ll} - C_{HWB} \frac{4 \hat{m}_W}{\sqrt{\hat{m}_Z^2 - \hat{m}_W^2}} \right), \quad (22)$$

$$\delta g_1^Z = \frac{1}{4\sqrt{2} \hat{G}_F} \left( C_{HD} - 4 C_{H\ell}^{(3)} + 2 C'_{ll} + 4 \frac{\hat{m}_Z}{\hat{m}_W} \sqrt{1 - \frac{\hat{m}_W^2}{\hat{m}_Z^2}} C_{HWB} \right), \quad (23)$$

$$\delta \kappa_\gamma = \frac{1}{4\sqrt{2} \hat{G}_F} \left( C_{HD} \frac{\hat{m}_W^2}{\hat{m}_W^2 - \hat{m}_Z^2} - 4 C_{H\ell}^{(3)} + 2 C'_{ll} \right), \quad (24)$$

$$\delta \kappa_Z = \frac{1}{4\sqrt{2} \hat{G}_F} \left( C_{HD} - 4 C_{H\ell}^{(3)} + 2 C'_{ll} \right), \quad (25)$$

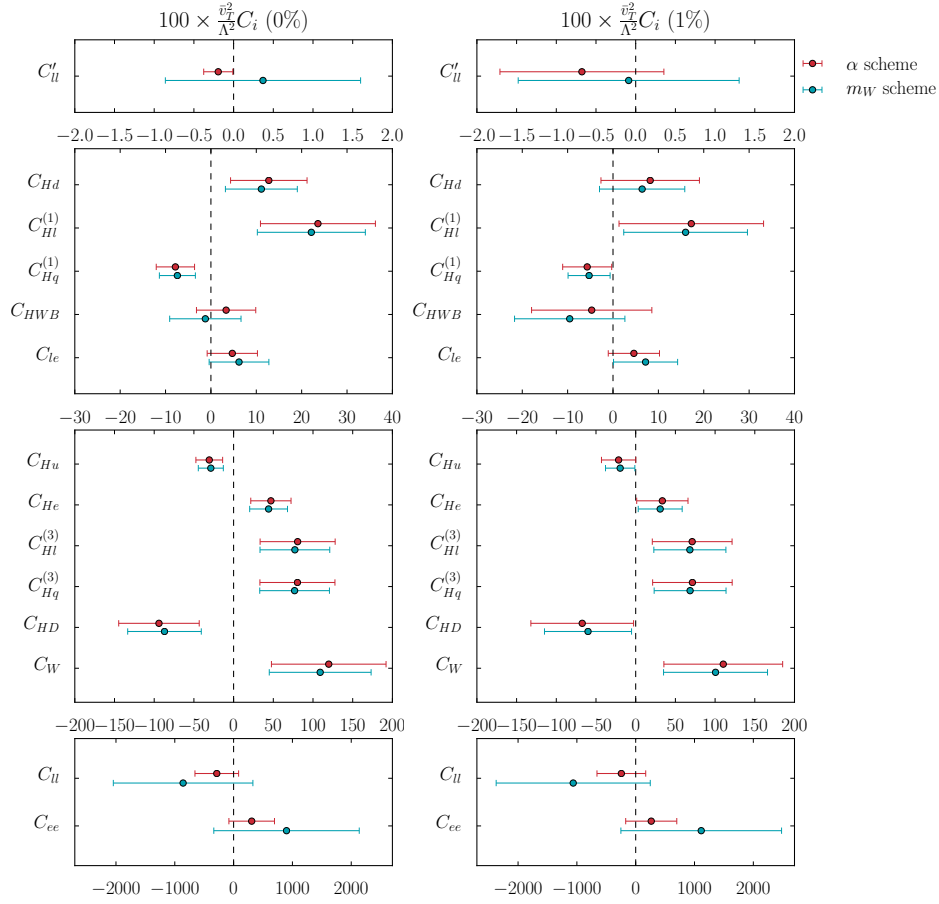
The list in eq. (3.37) should also include  $C'_{ll}$ :

$$\tilde{C}_i \equiv \frac{\bar{v}_T^2}{\Lambda^2} \{C_{He}, C_{Hu}, C_{Hd}, C_{Hl}^{(1)}, C_{Hl}^{(3)}, C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{HWB}, C_{HD}, C_{ll}, C'_{ll}, C_{ee}, C_{le}\}, \quad (37)$$

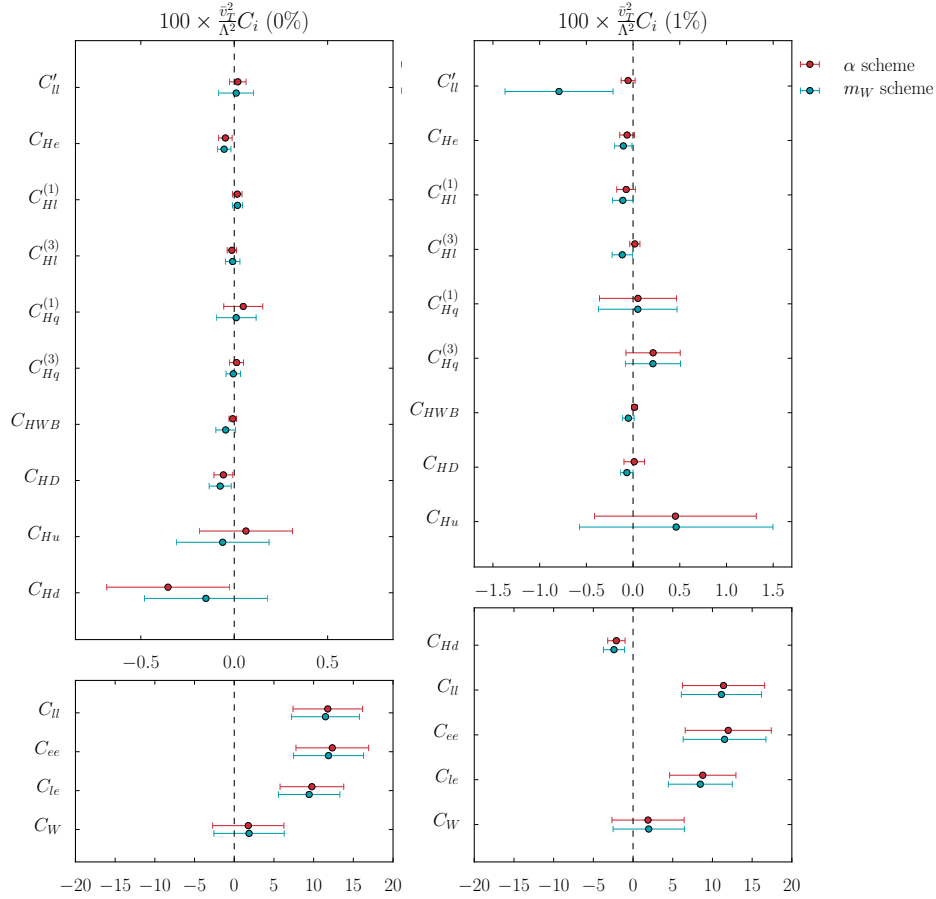
and the number of Wilson coefficients in the text after eq. (3.45) is then 21.

The fit results in this case are shown in figures 3, 4, 5 and tables 5, 6. The limits obtained minimizing the coefficients one-at-a-time are largely unchanged, while the fit results that marginalize over the larger set of parameters are modified. A significant scheme dependence is found for  $C'_{ll}$  in this case. This coefficient enters the considered observables via shift parameters. In the  $\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ -scheme it impacts most LEPI data, and in particular  $\hat{m}_W$ . In the  $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ -scheme it affects dominantly bhabha scattering via  $\delta\alpha$ , that is less constraining.  $C_{ll}$  and  $C_{ee}$  are poorly constrained and strongly anti-correlated as they both contribute to bhabha scattering only, where they enter in a linear combination of the form<sup>1</sup>  $[C_{ee} + (1 + \Delta(s, c_\theta)) C_{ll}]$  where  $0 < \Delta(s, c_\theta) < 0.1$  at the LEP2 c.m.s. energy. The direction  $C_{ll} - C_{ee}$  is nearly unconstrained and this degeneracy is weakly broken by the kinematic dependence. The correlations are larger in the  $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$  scheme for the observables considered.  $C'_{ll}$  is more correlated with  $C_{ll}$ ,  $C_{ee}$ ,  $C_{le}$  as bhabha scattering provides the dominant constraint on  $C'_{ll}$  in this scheme increasing correlations. In the  $\{\hat{\alpha}_W, \hat{m}_Z, \hat{G}_F\}$  scheme,  $C'_{ll}$  is primarily bounded by the  $m_W$  measurement, and this allows the parameters to split in less correlated blocks, one constrained by LEPI + WW production data and one by bhabha scattering.

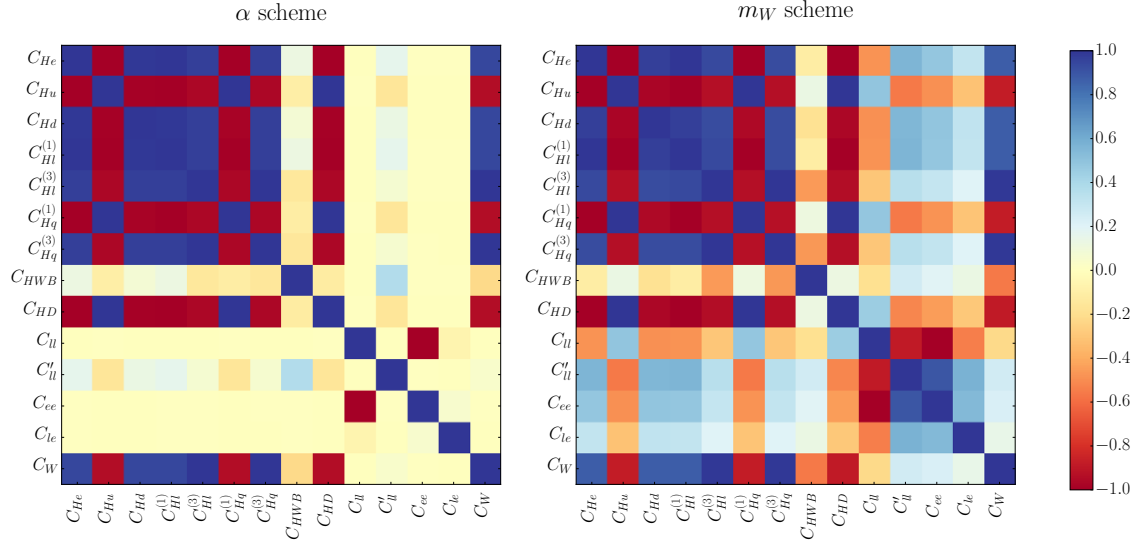
<sup>1</sup>Here  $c_\theta$  is the cosine of the angle between the incoming  $e^-$  and the outgoing  $e^+$  in bhabha scattering.



**Figure 3.** Best fit values of the Wilson coefficients (scaled by a factor 100) and corresponding  $\pm 1\sigma$  confidence regions obtained after profiling away the other parameters. Red (blue) points were obtained in the  $\{\hat{\alpha}(\hat{m}_W), \hat{m}_Z, \hat{G}_F\}$  input parameter scheme. The plot to the left has been obtained assuming  $\Delta_{\text{SMEFT}} = 0$ , while the one to the right includes a theoretical error  $\Delta_{\text{SMEFT}} = 0.01$ .



**Figure 4.** Best fit values of the Wilson coefficients (scaled by a factor 100) and corresponding  $\pm 1\sigma$  confidence regions obtained minimizing the  $\Delta\chi^2$  with one parameter at a time. Red (blue) points were obtained in the  $\{\hat{\alpha}(\hat{m}_W), \hat{m}_Z, \hat{G}_F\}$  input parameter scheme. The plot to the left has been obtained assuming  $\Delta_{\text{SMEFT}} = 0$ , while the one to the right includes a theoretical error  $\Delta_{\text{SMEFT}} = 0.01$ . Note that in the right plot the  $x$  axis has been scaled by a factor 2 and the coefficient  $C_{Hd}$  has been moved to the lower panel: increasing the theoretical error enhances the pull of the  $\mathcal{A}_{FB}^{0,b}$  anomaly compared to  $Z$  width data, and this relaxes by one order of magnitude the bound on this parameter.



**Figure 5.** Color map of the correlation matrix among the Wilson coefficients, obtained assuming zero SMEFT error, for the  $\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$  input scheme (left) and for the  $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$  input scheme (right).

$C_i \times \frac{\bar{v}_T^2}{\Lambda^2}$	$\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme		$\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ scheme	
	(0%)	(1%)	(0%)	(1%)
$C_{He}$	$47. \pm 25.$	$34. \pm 32.$	$44. \pm 24.$	$31. \pm 28.$
$C_{Hu}$	$-31. \pm 17.$	$-22. \pm 22.$	$-29. \pm 16.$	$-20. \pm 18.$
$C_{Hd}$	$12.8 \pm 8.4$	$8. \pm 11.$	$11. \pm 7.9$	$6.4 \pm 9.4$
$C_{Hl}^{(1)}$	$24. \pm 13.$	$17. \pm 16.$	$22. \pm 12.$	$16. \pm 14.$
$C_{Hl}^{(3)}$	$81. \pm 47.$	$71. \pm 50.$	$77. \pm 44.$	$68. \pm 45.$
$C_{Hq}^{(1)}$	$-7.8 \pm 4.2$	$-5.7 \pm 5.4$	$-7.4 \pm 4.0$	$-5.2 \pm 4.6$
$C_{Hq}^{(3)}$	$80. \pm 47.$	$71. \pm 50.$	$77. \pm 44.$	$69. \pm 45.$
$C_{HWB}$	$3.4 \pm 6.5$	$-5. \pm 13.$	$-1.2 \pm 7.9$	$-10. \pm 12.$
$C_{HD}$	$-94. \pm 51.$	$-67. \pm 65.$	$-87. \pm 46.$	$-60. \pm 55.$
$C_l$	$-286. \pm 371.$	$-244. \pm 414.$	$-859. \pm 1190.$	$-1062. \pm 1310.$
$C'_l$	$-0.19 \pm 0.18$	$-0.7 \pm 1.0$	$-0.37 \pm 1.2$	$-0.08 \pm 1.4$
$C_{ee}$	$308. \pm 388.$	$264. \pm 434.$	$890. \pm 1240.$	$1114. \pm 1366.$
$C_{le}$	$4.7 \pm 5.5$	$4.6 \pm 5.6$	$6.2 \pm 6.6$	$7.1 \pm 7.1$
$C_W$	$120. \pm 72.$	$110. \pm 75.$	$109. \pm 64.$	$101. \pm 65.$

**Table 5.** Best fit values and corresponding  $1\sigma$  confidence regions for  $\Delta_{\text{SMEFT}} = \{0\%, 1\%\}$  and for the two input parameter schemes considered in this work. The numbers have been obtaining after profiling the  $\chi^2$  over the other parameters and they have been multiplied by a factor 100.

$C_i \times \frac{\bar{v}_T^2}{\Lambda^2}$	$\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme		$\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ scheme	
	(0%)	(1%)	(0%)	(1%)
$C_{He}$	$-0.047 \pm 0.036$	$-0.064 \pm 0.079$	$-0.054 \pm 0.037$	$-0.104 \pm 0.092$
$C_{Hu}$	$0.06 \pm 0.25$	$0.45 \pm 0.87$	$-0.06 \pm 0.25$	$0.462 \pm 1.036$
$C_{Hd}$	$-0.35 \pm 0.33$	$-2.1 \pm 1.1$	$-0.152 \pm 0.33$	$-2.4 \pm 1.3$
$C_{Hl}^{(1)}$	$0.016 \pm 0.025$	$-0.07 \pm 0.10$	$0.018 \pm 0.026$	$-0.109 \pm 0.11$
$C_{Hl}^{(3)}$	$-0.013 \pm 0.025$	$0.019 \pm 0.054$	$-0.009 \pm 0.039$	$-0.12 \pm 0.11$
$C_{Hq}^{(1)}$	$0.05 \pm 0.10$	$0.05 \pm 0.41$	$0.01 \pm 0.11$	$0.05 \pm 0.42$
$C_{Hq}^{(3)}$	$0.013 \pm 0.037$	$0.21 \pm 0.29$	$-0.005 \pm 0.039$	$0.21 \pm 0.30$
$C_{HWB}$	$-0.008 \pm 0.020$	$0.015 \pm 0.029$	$-0.046 \pm 0.053$	$-0.050 \pm 0.061$
$C_{HD}$	$-0.058 \pm 0.051$	$0.01 \pm 0.11$	$-0.075 \pm 0.059$	$-0.066 \pm 0.066$
$C_{ll}$	$11.8 \pm 4.4$	$11.4 \pm 5.2$	$11.9 \pm 4.4$	$11.1 \pm 5.0$
$C'_{ll}$	$0.019 \pm 0.044$	$-0.053 \pm 0.074$	$0.011 \pm 0.094$	$-0.79 \pm 0.58$
$C_{ee}$	$12.4 \pm 4.6$	$12.0 \pm 5.4$	$11.9 \pm 4.4$	$11.5 \pm 5.2$
$C_{le}$	$9.8 \pm 4.0$	$8.8 \pm 4.2$	$9.4 \pm 3.9$	$8.5 \pm 4.0$
$C_W$	$1.8 \pm 4.5$	$1.9 \pm 4.5$	$1.9 \pm 4.4$	$2.0 \pm 4.5$

**Table 6.** Best fit values and corresponding  $1\sigma$  confidence regions for  $\Delta_{\text{SMEFT}} = \{0\%, 1\%\}$  and for the two input parameter schemes considered in this work. These numbers have been obtained minimizing the  $\chi^2$  with one parameter at a time (despite the non-minimal character of the SMEFT [1]), and they have been multiplied by a factor 100.

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